SE/IV/COMP-IT-CSE-AI&DS/C-Scheme/DEC-24/02-12-24.

(Time : 3 Hours) **(Total Marks : 80)**

Note :

1) **Q. No. 01 is compulsory.**
2) **Solve any three from Q. No. 02 to 06.**
3) **Numbers to the right indicate full marks.**
4) **Use of statistical tables is allowed.**

Q. 1. Solve.

a) If $A = \begin{bmatrix} -1 & 2 & 38 \\ 0 & 2 & 37 \\ 0 & 0 & -2 \end{bmatrix}$ find the Eigen values of $A^3 + 5A + 8I$. **05**

b) Integrate the function $f(z) = x^2 + i\,xy$ from A(1, 1) to B(2, 4) along $y = x^2$ **05**

c) Find the Z-Transform of $f(k) = a^{-k}$, $k \geq 0$. **05**

d) If a random variable X follows Poisson distribution such that P (x = 1) = 2 P (x = 2). Find mean and variance of the distribution. **05**

Q. 2.

a) Find the Eigenvalues and Eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **06**

b) Find the Z-Transform of $\cos\left(\frac{\pi}{4} + k\alpha\right)$ $k \geq 0$. **06**

c) Use the dual simplex method to solve the LPP **08**
Min. $Z = 2X_1 - X_2 + 3X_3$,
$3X_1 - X_2 + 3X_3 \leq 7,\ 2X_1 - 4X_2 \geq 12, X_1, X_2, X_3, \geq 0$

Q. 3.

a) Evaluate $\int_C \frac{z+8}{z^2+5z+6} dz$ Where C is a circle $|z|=5$. **06**

b) Verify Caley-Hamilton theorem and hence find A^{-1} and A^4 where $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. **06**

c) Solve the LPP by Big -M method **08**
Max. $Z = X_1 + 2X_2 + 3X_3 - X_4$
$X_1 + 2X_2 + 3X_3 = 15$, $2X_1 + X_2 + 5X_3 = 20$, $X_1 + 2X_2 + X_3 + X_4 = 10$ $X_1, X_2, X_3, X_4 \geq 0$

Q. 4.

a) Find inverse Z transform of $F(z) = \frac{1}{(z-2)(z-3)}$ for i) $|z| < 2$, ii) $|z| > 3$. **06**

b) A certain drug administered to 12 patients resulted in the following change in their blood pressure. 5, 2, 8, - 1, 3, 0, 6, - 2, 1, 0, 4,5 Can we conclude that the drug increases the blood pressure? **06**

c) Find all possible Laurent's series expansions of the function $f(z) = \frac{1}{(z+1)(z-2)}$ about $z = 0$ indicating the region of convergence in each case. **08**

Q. 5.

a) Determine all basic solutions to the following problem **06**

Max $= x_1 - 2x_2 + 4x_3$,

$x_1 + 2x_2 + 3x_3 = 7,\ 3x_1 + 4x_2 + 6x_3 = 15\ , x_1\ , x_2, x_3 \geq 0$.

b) If X is a Normal variate with mean 10 & s.d. 4, find i) P(5≤X≤18), ii) P(X≤12). **06**

c) Solve the NLPP **08**

Optimize $Z = 12x_1 + 8x_2 + 6x_3 - {x_1}^2 - {x_2}^2 - {x_3}^2 - 23$

Subject to $x_1 + x_2 + x_3 = 10\ , x_1\ , x_2\ ,\ x_3 \geq 0$.

Q. 6.

a) Show that the given matrix is diagonalizable and hence find diagonal form and transforming matrix where $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **06**

b) Based on the following data if there is a relation between literacy and smoking. **06**

	Smoking	Non-smoking
Literacy	83	57
Illiteracy	45	68

c) Max. $Z = 12x_1x_2 + 2{x_1}^2 - 7{x_2}^2$, Subject to $2x_1 + 5x_2 \leq 98, x_1\ , x_2 \geq 0$ by K-T condition. **08**

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