

(Duration: 3 hours)

Max.Marks:80

- N.B:** (1) Question No.1 is compulsory
 (2) Answer any three questions from Q.2 to Q.6
 (3) Figures to the right indicate full marks.

- 1 a) Prove that $\log \left(\frac{2+3i}{2-3i} \right) = 2i \tan^{-1} \left(\frac{3}{2} \right)$ 5
- b) Prove that every square matrix can be uniquely expressed as sum Hermitian and skew Hermitian matrix. 5
- c) If $z = x^2y + y^2$, $x = \log t$, $y = e^t$, find $\frac{dz}{dt}$ at $t = 1$. 5
- d) Find the n^{th} derivative of $\frac{x}{(2x+3)(x+2)}$ 5
- 2 a) Prove that $\sin^5 \theta = \frac{1}{16} (\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta)$ 6
- b) If $u = f \left(\frac{y-x}{xy}, \frac{z-x}{xz} \right)$, then show that 6
- $$x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$$
- c) Test for consistency the following system & solve them if consistent 8
- $$x_1 - 2x_2 + x_3 - x_4 = 2, \quad x_1 + 2x_2 + 2x_4 = 1, \quad 4x_2 - x_3 + 3x_4 = -1$$
- 3 a) Prove that $(1 + i\sqrt{3})^n + (1 - i\sqrt{3})^n = 2^{n+1} \cos \frac{n\pi}{3}$ 6
- b) Find the extreme values of the function $x^2y - 3x^2 - 2y^2 - 4y + 3$ 6
- c) Find the real root of $x^3 - 2x - 5 = 0$ correct up to three places of decimal using Newton-Raphson Method. 8

- 4 a) If $x + iy = \cot\left(\frac{\pi}{6} + i\alpha\right)$ P.T $x^2 + y^2 - 2\frac{x}{\sqrt{3}} = 1$ 6
- b) Expand $\tan^{-1}(x)$ in powers of $(x - \frac{\pi}{4})$. 6
- c) If $\cos^{-1}\frac{y}{b} = \log\left(\frac{x}{n}\right)^n$, then prove that 8
- $$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$$
- 5 a) Separate real and imaginary parts of $(1+i)^{1-i}$ 6
- b) Solve the following equations by Gauss Jacobi's Iteration method: 6
- $$15x + 2y + z = 18, 2x + 20y - 3z = 19, 3x - 6y + 25z = 22.$$
- c) Solve: $x^{10} + 8x^5 + 15 = 0$. 8
- 6 a) If $u = \tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ Prove that 6
- $$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -2\sin^3 u \cos u$$
- b) i) Prove that $\sinh^{-1}(\tan \theta) = \log\left[\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]$ 6
- ii) If $u = \log\left(\frac{x}{y}\right)$, then find $xu_x + yu_y$
- c) If $A = \begin{pmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{pmatrix}$ find non-singular matrices P and Q such that PAQ is in normal form and find its rank. 8