

(Time: 3 hours)

Max.Marks:80

- N.B (1) Question No.1 is compulsory
(2) Answer any three questions from Q.2 to Q.6
(3) Figures to the right indicate full marks.

- 1 (a) If $\tan(\alpha + i\beta) = x + iy$ then show that $\tanh 2\beta = \frac{2y}{1+x^2+y^2}$ 5
- (b) If $z = \sin^{-1}(x-y)$, $x=3t$, $y=4t^3$ prove that $\frac{dz}{dt} = \frac{3}{\sqrt{1-t^2}}$ 5
- (c) If $y = x^2 \sin x$, prove that 5
- $$y_n = (x^2 - n^2 + n) \sin\left(x + \frac{n\pi}{2}\right) - 2nx \cos\left(x + \frac{n\pi}{2}\right)$$
- (d) Find the real root of the equation $x^3 - 2x - 5 = 0$ by Newton-Raphson method, correct to three places of decimals. 5
- 2 (a) Find k such that the following system of equations has 6
- (1) Unique solution (2) many solutions (3) no solution.
- $$kx + y + z = 1, x + ky + z = 1, x + y + kz = 1$$
- (b) Solve $x^6 + 1 = 0$ using De Moivre's theorem. 6
- (c) Solve by Gauss-Seidel method with an accuracy of 0.0001 8
- $$5x + y - z = 10; 2x + 4y + z = 14; x + y + 8z = 20$$
- (5 iterations only)
- 3 (a) Solve the equations $x_1 + x_2 - x_3 + x_4 = 0$ 6
- $$x_1 - x_2 + 2x_3 - x_4 = 0 \quad 3x_1 + x_2 + x_4 = 0$$
- (b) Prove that $\left[\frac{1 + \sin\left(\frac{\pi}{8}\right) + i \cos\left(\frac{\pi}{8}\right)}{1 + \sin\left(\frac{\pi}{8}\right) - i \cos\left(\frac{\pi}{8}\right)} \right]^8 = -1$ 6

- (c) If $u = \frac{x^2 y^2}{x^2 + y^2} + \cos^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$ find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ using Euler's theorem. 8
- 4 (a) Prove that $A - A^o$ is skew Hermitian where 6

$$A = \begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$$
- (b) Find the extreme values of $f(x, y) = xy(3 - x - y)$ 6
- (c) Show that $\frac{\sin 6\theta}{\sin 2\theta} = 16 \cos^4 \theta - 16 \cos^2 \theta + 3$ 8
- 5 (a) Expand in powers of x using Maclaurin's series and find the values of a, b, c where $\log \sec x = ax^2 + b\frac{x^4}{4} + c\frac{x^6}{6} + \dots$ 6
- (b) If $u = f(e^{x-y}, e^{y-z}, e^{z-x})$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ 6
- (c) If $\tan(\theta + i\phi) = \tan \alpha + i \sec \alpha$, then show that 8

$$e^{2\phi} = \cot \frac{\alpha}{2}, 2\theta = n\pi + \frac{\pi}{2} + \alpha.$$
- 6 (a) Find non-singular matrices P and Q such that PAQ is in the normal form of A . Hence find rank of A where $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}$ 6
- (b) Show that $\log(-\log i) = \log \frac{\pi}{2} - i \frac{\pi}{2}$ 6
- (c) If $y = \frac{x}{x^2 + a^2}$ prove that $y_n = \frac{(-1)^n n! \sin^{(n-1)} \theta}{a^{(n+1)}} \cos(n+1)$ where 8

$$\theta = \tan^{-1} \left(\frac{a}{x} \right)$$